CONCEPTUAL APPROACHES TO THE ASSESSMENT OF THE ECONOMIC SYSTEMS' SENSITIVITY

A. Kibitkin and N. Nedel'ko
Murmansk State Technical University
Murmansk, Russia

The problem of the interaction between enterprises and external and internal environment, as well as the development of mechanisms that provide the effectiveness of this interaction is of particular significance and relevance.

Consequently, it is necessary to apply more advanced approaches to strategy and planning system of the industrial and economic activity of an enterprise, to create more effective management.

At the present time relevant to the works on economic, managerial problems is a systematic approach, one of the basic concepts of which is the notion of system.

Economic system is determined as a set of interconnected economic elements that form the resistant integrity.

The economic system's functioning represents the set of coordinated actions which are necessary for the carrying out of a definite task.

In the analysis of economic system its sensitivity to the influence of various parameters (or factors): technical, manufacturing, economic, financial, plays an important role.

Consider the influence of the factors of external environment on the economic system and its possible response in connection with the change of main properties, one of which is the stability.

Suppose that the system resides in initial state at the moment of time t_0.

External factors affect the properties of the system and change them with a certain force.

In the process of influence every factor can:

1. intensify the action of the inner property of the system with the endless speed – namely there is a jump, or with definite speed – namely arises the smooth transition of the system in another state;

2. decrease the action of the inner property of system with endless or with definite speed;

3. extinguish acting process in the economic system;

4. does not act on the properties of the system, that is to pass through the system in the same state, as on entrance or to be extinguished in it

The new state of the system – the state, in which she passed changing its properties under the influence of external parameters – can be stable, unstable or the system can reside in the interim area.

For ensuring of the resistant development of economic system or for transition from one state to other on this stage is apply fitting sequence of management tools.

The influence of the management tools can return system to the state, close to the initial one or to transfer it to another area of stability. Following variants are possible here:

1. If the system resided in the resistant area, then arises its further development, or system can lose stability and turn into the interim area;

2. If the system was in the interim area, then influence of the management tools can transfer system to the resistant or unsteady area;

3. If the system resided in unsteady area, then influence of the management tools can transfer system to the interim area or lead to the business failure.

Since the economic system isn't only by influenced control tools but also exterior parameters which change the properties of the system with various force and the behavior of its elements the any reaction of system on the control tools is possible – either positive or negative. That is why it is necessary to research the:

1. the initial state of the system at the moment of time t_0;

2. reactions of the system for the exterior perturbation actions, the research of the sensitivity of the system to the action of various parameters;

3. the behavior of elements and changes of the main properties of the system;

4. also it is necessary to determine the location of the system in one of the region of stability and to forecast the further behavior of the system (the transition in which area and for what time) on the basis of the sensitivity.

5. in order to optimize the behavior of economic system and its development the set of the management tools is chosen.

It is assumed, that the strength of the set of tools to achieve a positive result must exceed the force of the influence of negative factors of exterior environment, namely it means that it is necessary to choose such instruments that: Extinguish the action of one parameters; Enlarge the action of other parameters; Act on economic system by developing its property and optimizing the objective function.

Thus, the sensitivity of the economic system can be determined as property, the reaction of the system to the action of parameters, the speed of the change of system behavior depending on the parameter variations of and exterior perturbation influences.

By analogy with determining of the threshold of sensitivity in physiology and technique, wel determine the absolute and differential thresholds of sensitivity in the economic system.

The absolute threshold of sensitivity in the economic system – is the minimal force of the influence of the parameter (of indignation), capable to cause the system reaction.

Differential threshold – is the minimal value, on which it is necessary to change the influence of parameter, in order to cause the change of the reaction of system.

We can distinguish the main conceptual approaches to the sensitivity assessment.

The first approach is based on determining the sensitivity of automatic control systems, sensitivity functions.

General model of continuous dynamic systems – the vector differential equation of the nonlinear system:

$$\dot{\boldsymbol{x}} = f\left(\boldsymbol{x}(t), \boldsymbol{p}, t\right), \ \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \ (1)$$

where x – is the vector of state; p – is the vector of parameters; f – is the vector function that connects the vector of state and the vector of the parameters of the system.

Since economic system is continuous dynamic system that its behavior maybe described with the help of vector differential equation (1).

For small variations of parameters the additional movement may be described by factorizing expansion terms (1) in Taylor' series. Confining to linear expansion terms, additional movement can be expressed in the following way:

$$\Delta \boldsymbol{x}(t) = \sum_{j=1}^{l} \varepsilon_j \Delta p_j$$

where $\boldsymbol{\varepsilon}_j$ – is the vector of the sensitivity of the vector of state $\mathbf{x}(t)$; $\mathbf{x}_Д(t)$ – is the valid motion of system; $\mathbf{x}_H(t)$ – is the nominal motion of system.

Partial derivative, which are determined in the nominal significances of parameters, are named sensitivity index or function of the vector of state υ_j

$$\upsilon_j = \left[\frac{\partial x_1}{\partial p_j}, ..., \frac{\partial x_n}{\partial p_j}\right]^T = \frac{\partial \boldsymbol{x}}{\partial p_j}, \ j = \overline{1,l}$$

here is component of vector characterizes the rate of change of the element of state vector $\boldsymbol{x}(t)$ if we change the j-th component of the vector of parameters $\boldsymbol{p}$.

The summary systems reaction on the parameter variation is well captured by the norm of the vector which are widely used in the analysis of dynamical systems.

For continuous systems in determining the sensitivity functions there are two cases: changed the parameter variation in time or not ($\Delta p = \varepsilon$ =const or $\Delta p = \varepsilon \upsilon(t)$).

If the variation of the parameter does not depend on time, the function of the sensitivity of the system $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{p}, t)$ is determined by the ordinary derivative:

$$u(p_i) = \frac{\partial x}{\partial p_i} = \lim_{\varepsilon \to 0} \frac{x(p_H + \varepsilon) - x(p_H)}{\varepsilon}$$

If the perturbation depends on time, the sensitivity function should be defined as the functional derivative:

$$u(p_i) = \frac{\partial x}{\partial p_i} = \lim_{\varepsilon \to 0} \frac{x(p_H + \varepsilon \upsilon(t)) - x(p_H)}{\varepsilon}$$

where – uniformly limited, integrable function.

The economic system is a continuous system, that is why consideration of the definition of sensitivity to it is applicable.

The second approach is based on determining the sensitivity functions via transfer functions.

The transfer function of the dynamic system (or element) in the control theory is defined as the function that determines the relationship between input and output values.

$$V_{W_\delta}^K(p) = \frac{dK(p)}{dW_\delta(p)}$$

where $K(p)$ – transfer function of system; $W_\delta(p)$ – transfer function of a variable element.

Sensitivity $S_{W_\delta}^K(p)$ is the ratio of relative changes in the transfer function of system and transfer function of a variable element:

$$S_{W_\delta}^K(p) = \frac{dK(p)}{K(p)} \Big/ \frac{dW_\delta(p)}{W_\delta(p)} = V_{W_\delta}^K(p)\frac{W_\delta(p)}{K(p)}$$

The lower the sensitivity $S_{W_\delta}^K(p)$ or function of sensitivity $V_{W_\delta}^K(p)$, that less effect of the transfer function $W_\delta(p)$ elements on the properties of the system.

Talking about the change of sensitivity, we mean the change of its modulus, and therefore of great interest are the structures which have a low sensitivity. The lower the sensitivity of the system, the more it is high-quality.

For economic systems as elements can be considered internal departments, and as the connections of elements - the interaction of these units with each other (management industry, business, service, shop and so on down the levels of management).

The third approach, we consider the sensitivity as elasticity.

Since the coefficient of elasticity is the sensitivity, then this feature of the economic system can be determined from this point of view.

The sensitivity of the objective function (profit, work efficiency) of the parameter indicates the percentage change in the function as the parameter change by 1%.

$$\varepsilon = \frac{dF}{dp}$$

The relative change of the function is equal to the change of function to its value. The relative change of the parameter is equal to the change of parameter to its value.

Thus, $\varepsilon = \frac{dF}{dp} = \frac{\Delta F \cdot p}{F \cdot \Delta p} = \frac{\Delta F}{\Delta p} \cdot \frac{p}{F}$.

Sensitivity will show the rate of change of the objective function of the system (for example, profit), depending on the external parameters. At any time of the production

process sensitivity is determined as $\varepsilon(t) = \frac{dF}{dp}(t)$.

According to the definition of elasticity, we can distinguish three ranges of sensitivity of the economic system (in absolute value):

1. $(0 < \varepsilon < 1)$: at $\varepsilon \to 0$ infinite variation of parameter $\Delta p \to \infty$ does not affect the value of the function, i.e. it will remain constant;

2. $(1 < \varepsilon < \infty)$: at $\varepsilon \to \infty$, any small decrease of parameter $(\Delta p \to 0)$function will increase to infinity (there is a sharp change in the function), and any small increase in the parameter lead to a decrease in function to zero;

3. at $\varepsilon = 1$ and in some neighborhood of «1» can be observed the function change at 1% in the parameter change by 1%.

Concluding the review of basic concepts of the theory of sensitivity, it should be noted that this theory and its methods can't be considered exhausted: the study of the sensitivity of the economic system is in a state of continuous development, supplemented by new theoretical and experimental results.

The work was submitted to the International Scientific Conference «Fundamental research», Israel (Tel Aviv), April, 10-17, 2010. Recieved by editorial office on 03.04.2010.